10

15

20

25

Transmission system using an improved signal encoder and decoder.

The present invention is related to a transmission system comprising a transmitter with a signal encoder having an input for a signal to be encoded, said signal encoder comprises a codebook entry selector for selecting a codebook entry for obtaining a synthetic signal giving a best approximation of a signal representative of the input signal, the codebook entry comprises a plurality of samples that can assume more than two values, said codebook entry being identified with a sequence of symbols, the transmitter being arranged for transmitting the sequence of symbols to a receiver, the receiver comprises a decoder with a codebook for deriving the codebook entry from the received sequence of symbols.

A prior art transmission system is known from the conference paper "An algorithm for assigning binary indices to the code vectors of a multi-dimensional quantizer" by J. De Marca and N. Jayant published in the proceedings of the IEEE International Conference on Communications '87(ICC-87), Volume 2, pp. 1128-1132.

Such transmission systems are e.g. used in applications in which speech or video signals have to be transmitted over a transmission medium with a limited transmission capacity or have to be stored on storage media with a limited storage capacity. Examples of such applications are the transmission of speech signals over the Internet, the transmission of speech signals from a mobile phone to a base station and vice versa and storage of speech signals on a CD-ROM, in a solid state memory or on a hard disk drive.

In a transmission system according to the preamble, the signal to be encoded is compared with a plurality of synthetic signal segments. Each of the synthetic signal segments is derived from one of the codebook entries. The synthetic signal segments can e.g. be obtained by filtering the sequence of samples contained in the codebook entry by means of a synthesis filter. The codebook entry corresponding to the synthetic signal segment which best matches the input signal is encoded and transmitted to the receiver.

An alternative possibility is to derive a residual signal from the input signal by means of an analysis filter and to compare the residual signal with each of the codebook

10

15

entries. The codebook entry best matching the residual signal is encoded and transmitted to the receiver.

It is also conceivable that the input signal is directly compared with the codebook entries and that the best matching codebook entry is encoded and transmitted.

In the receiver, the received code associated with the codebook entry is decoded and a replica of the input signal is reconstructed. This can be done by applying the plurality of samples to a synthesis filter which has a similar transfer function as the synthesis filter used in the encoder. If an analysis filter is used in the encoder, a synthesis filter is used which has a transfer function which is the inverse of the transfer function of the analysis filter.

If no analysis or synthesis filter is used in the encoder, the reconstructed signal is directly derived from the decoded codebook entry.

It can happen that due to transmission impairments, the encoded codebook entry is received in error. Consequently, in the receiver a codebook entry different from the codebook entry selected in the encoder will be used for reconstructing the input signal. Using the wrong codebook entry for reconstructing the input signal will in general result in an audible/visible error in the reconstructed signal.

In the transmission system according to the above mentioned conference paper it is tried to minimize the effect of transmission errors by assigning to similar codebook entries similar sequences of symbols in such a way that if a transmission error occurs in one of the symbols, the codebook entry corresponding to said erroneously received sequence of symbols differs only slightly from the codebook entry corresponding to the originally transmitted sequence of symbols. In this way it is obtained that the perceptual effect of a transmission error is substantially reduced.

The object of the present invention it to provide a transmission system in which the perceptual effect of transmission errors is even more reduced than in the prior art system.

To achieve said object the present invention is characterized in that the codebook entries corresponding to sequences of symbols differing in one particular symbol value, differ in one single sample value. This particular symbol value can be the least significant symbol, but it is also possible that it is a symbol at a different position in the sequence of symbols.

For the purpose of designing the assignment of sequences of symbols to codebook entries in the prior art system, it is assumed that every symbol in the sequence of

25

30

20

symbols can be in error. This assumption results in a non-optimum assignment of codebook entries to sequences of symbols when it is taken into account that the possibility of a transmission error often differs for several symbols. It is possible that an error correcting code is used for a part of the sequence of symbols. It is also possible that hierarchical modulation is used resulting in different error probabilities. By restricting the number of symbols which can be in error, it becomes possible to reduce the difference between the codebook entries.

By making codebook entries differing in one single sample to correspond to sequences of symbols differing in one particular symbol value (mostly the most vulnerable one) a near optimum codebook is obtained.

An embodiment of the present invention is characterized in that the difference between said sample values of codebook entries corresponding to sequences of symbols differing in one particular symbol value, is equal to a smallest quantization step of said sample value.

By choosing the difference between the sample values corresponding to "neighboring" sequences of symbols equal to the smallest quantization step, an optimum codebook with respect to the perceptual effect of a single transmission error is obtained.

A further embodiment of the invention is characterized in that the number of possible sample values is odd. It is found that in the case of an odd number of possible values it becomes possible to calculate the mapping between sequences of symbols and the corresponding plurality of samples and its inverse with the same algorithm. This results in a reduced amount of resources required to implement a combination of encoder and decoder, because the resources for performing the codebook related calculation can be shared.

If the combination of encoder and decoder is realized by a program running on a programmable processor, the amount of memory to hold the program is reduced. If the combination of encoder and decoder is realized in hardware, the amount of chip area will be reduced because the part for determining the sequence of symbols from the plurality of samples can also be used for determining the plurality of samples from the sequence of symbols.

A still further embodiment of the present invention is characterized in that a numerical value associated with a first codebook entry is equal to the numerical value of the sequence of symbols of a second codebook entry, and in that the numerical value associated with the second codebook entry is equal to the numerical value of the sequence of symbols associated with the first codebook entry.

10

15

20

25

30

of the transfer of the transfe

According to this aspect of the invention, it becomes possible to determine the index of a given codebook entry by first using said given codebook entry as index to determine a second codebook entry and secondly by using the second codebook entry as index to determine a codebook entry which represents the index of the given codebook entry.

5

The invention will now be explained with reference to the drawings.

- Fig. 1 shows a transmission system in which the present invention can be used.
- Fig. 2 shows a speech encoder according to the invention.
- Fig. 3 shows a speech decoder according to the invention.

Fig. 4 shows a flow graph of a program for a programmable processor for converting a sequence of symbols indicating the codebook index into the corresponding plurality of samples.

15

10

In the transmission system according to Fig. 1 the signal to be transmitted is applied to a source encoder 4 in a transmitter 2. This source encoder 4 encodes the input signals using the present invention as will be explained later. The encoded signal available at the output of the source encoder 4 is applied to an input of a channel encoder 6. The channel encoder 6 encodes a part of the output signal of the source encoder.

20

For use of the present invention it is possible that all bits but one of the sequence of symbols indicating the codebook entry are encoded by the channel encoder 6. For mobile radio transmission systems often convolutional codes are used in the channel encoder 6.

25

The output of the channel encoder 6 is connected to the input of a modulator 8 which modulates the output signal of the channel encoder 6 onto a carrier. Subsequently the modulated signal is amplified and applied to an antenna 10.

30

It is observed that it is possible to apply hierarchical modulation to transmit the sequence of symbols corresponding to the codebook entries. The symbol which, when transmitted erroneously, gives the least perceptual effect is modulated on a sub-constellation which is superimposed on a main constellation. The remaining symbols of the sequence of symbols are modulated on the main constellation.

10

15

20

25

30

The sub-constellation has a smaller distance between its points than the distance between the points of the main constellation. Consequently, the symbols transmitted on the main constellation are less prone to errors than symbols modulated on the sub-constellation.

In a situation where hierarchical modulation is used it is conceivable that the channel encoder can be dispensed with.

The signal transmitted by the antenna 10 is received by the antenna 12 and is passed to the receiver 14. In the receiver 14 the antenna signal is demodulated in a demodulator 16. The demodulator 16 passes the demodulated signal to a channel decoder 18. The channel decoder 18 decodes the received signals and corrects errors in them if possible. It is observed that it is possible that some symbols in the received signal are not encoded at all, and consequently they are passed to the output of the channel decoder unchanged. In the case that hierarchical modulation is used, it is also conceivable that the channel encoder 18 can be dispensed with. In the source decoder 20 the input signal of the transmitter 2 is reconstructed.

In the source encoder 4 according to Fig. 2 the signal to be encoded is applied to an input of an LPC coefficient calculation block 34 and to an input of a perceptual weighting filter 36. The output of the perceptual weighting filter 36 is connected to a first input of a subtractor 40.

An excitation signal generator 22 comprises a fixed codebook which is implemented as a ternary generator 26 and an adaptive codebook 24 in which the most recently used excitation signals are stored. The output signal of the ternary generator 26 represents a plurality of ternary samples, in which each digit of the ternary number represents a ternary sample value.

The output of the ternary generator 26 is connected to an input of a code converter 29 which is arranged for converting the ternary value at the output of the ternary generator 26 into a sequence of (binary) symbols for transmission. The output of the ternary generator 26 is also connected to a first input of a multiplier 30, optionally via a zero inserter 27. A signal G_O is applied to a second input of the multiplier 30. The output of the multiplier 30 is connected to a first input of an adder 32.

The output of the adaptive codebook 24 is connected to a first input of a multiplier 28 and a signal G_A is applied to a second input of the multiplier 28. The output of the multiplier 28 is connected to a second input signal of the adder 32. The output of the adder 32 which constitutes also the output of the excitation signal generator 28 is applied to a perceptually weighted synthesis filter 38 which received its filter coefficients from the LPC

10

15

20

25

30

The sign with the first the sign of the si

coefficient calculating block 34. An output of the perceptually weighted synthesis filter 38 is connected to a second input of the subtractor 40.

The output of the subtractor 40 is connected to an input of a controller 42. The controller 42 is arranged for finding an excitation signal resulting in a best match between the perceptually weighted speech signal available at the output of the perceptual weighting filter 36 and the perceptually weighted synthetic speech signal which is available at the output of the perceptually weighted synthesis filter 38. The controller 42 first determines the codebook index I_A and the codebook gain G_A for the adaptive codebook. The adaptive codebook holds the excitation samples applied to the synthesis filter 38 from previous excitation intervals. Due to the periodicity of (voiced) speech signals, it is likely that the best sequence of excitation samples is similar to a sequence of excitation samples present in the adaptive codebook.

After the optimum parameters I_A and G_A have been found, the control means 42 continues with searching the optimum excitation parameters of the fixed codebook. The excitation parameters of the fixed codebook are the fixed codebook index I_F and the fixed codebook gain G_F . It is also possible that the excitation signal derived form the fixed codebook is constituted by a grid of excitation pulses having a plurality of excitation signal samples separated by a predetermined amount of zeros. In such a case also the position PH of the excitation samples in the grid has to be determined.

The search for the excitation parameters I_F and G_F is performed for each of the possible values of the position PH. The possible sequences of excitation samples are found by using a ternary generator 26 generating said ternary sequence of samples. For each sequence of (ternary) samples the optimum gain is determined. This gain can be determined by trying all possible gain values and selecting the value G_F which results in a minimum error between the perceptually weighted speech signal and the perceptually weighted synthetic speech signal. It is also possible to determine the gain factor G_F by first determining an auxiliary signal by subtracting from the perceptually weighted speech signal the contribution of the adaptive codebook to the perceptually weighted synthetic speech signal. The square of the gain factor G_F can be found by dividing the cross correlation coefficient of the auxiliary signal and a perceptually weighted synthetic speech signal which is subjected to a gain of 1, by the power of said perceptually weighted synthetic speech signal.

These ways of determining the gain factor G_F are well described in the prior art and are as such known to those skilled in the art.

7

In the table below a first example of a fixed codebook is given. In the table the binary sequence of symbols and the corresponding plurality of sample values is given. G(i) represents the sample value as a ternary number and E(i) represents the sample values as they are applied to the synthesis filter. In the codebook according to Table 1, the number of samples in one codebook entry equals to 3.

G(i)	E(i)	B(i)	G(i)	E(i)	B(i)	G(i)	E(i)
000	-1,-1,-1	01001	122	0, +1,+1	10010	200	+1,-1,-1
001	-1,-1, 0	01010	121	0, +1, 0	10011	201	+1,-1, 0
002	-1,-1,+1	01011	120	0, +1,-1	10100	202	+1,-1,+1
012	-1, 0,+1	01100	110	0, 0, -1	10101	212	+1, 0,+1
011	-1, 0, 0	01101	111	0, 0, 0	10110	211	+1, 0, 0
010	-1, 0,-1	01110	112	0, 0,+1	10111	210	+1, 0,-1
020	-1,+1,-1	01111	102	0, -1,+1	11000	220	+1,+1,-1
021	-1,+1, 0	10000	101	0, -1, 0	11001	221	+1,+1, 0
022	-1,+1,+1	10001	100	0, -1,-1	11010	222	+1,+1,+1
	000 001 002 012 011 010 020 021	000 -1,-1,-1 001 -1,-1,0 002 -1,-1,+1 012 -1,0,+1 011 -1,0,0 010 -1,0,-1 020 -1,+1,-1 021 -1,+1,0	000 -1,-1,-1 01001 001 -1,-1,0 01010 002 -1,-1,+1 01011 012 -1,0,+1 01100 011 -1,0,0 01101 010 -1,0,-1 01110 020 -1,+1,-1 01111 021 -1,+1,0 10000	000 -1,-1,-1 01001 122 001 -1,-1,0 01010 121 002 -1,-1,+1 01011 120 012 -1,0,+1 01100 110 011 -1,0,0 01101 111 010 -1,0,-1 01110 112 020 -1,+1,-1 01111 102 021 -1,+1,0 10000 101	000 -1,-1,-1 01001 122 0, +1,+1 001 -1,-1,0 01010 121 0, +1, 0 002 -1,-1,+1 01011 120 0, +1,-1 012 -1,0,+1 01100 110 0, 0, -1 011 -1,0,0 01101 111 0, 0, 0 010 -1,0,-1 01110 112 0, 0,+1 020 -1,+1,-1 01111 102 0, -1,+1 021 -1,+1,0 10000 101 0, -1, 0	000 -1,-1,-1 01001 122 0, +1,+1 10010 001 -1,-1, 0 01010 121 0, +1, 0 10011 002 -1,-1,+1 01011 120 0, +1,-1 10100 012 -1, 0,+1 01100 110 0, 0, -1 10101 011 -1, 0, 0 01101 111 0, 0, 0 10110 010 -1, 0,-1 01110 112 0, 0,+1 10111 020 -1,+1,-1 01111 102 0, -1,+1 11000 021 -1,+1,0 10000 101 0, -1, 0 11001	000 -1,-1,-1 01001 122 0, +1,+1 10010 200 001 -1,-1, 0 01010 121 0, +1, 0 10011 201 002 -1,-1,+1 01011 120 0, +1,-1 10100 202 012 -1, 0,+1 01100 110 0, 0, -1 10101 212 011 -1, 0, 0 01101 111 0, 0, 0 10110 211 010 -1, 0,-1 01110 112 0, 0,+1 10111 210 020 -1,+1,-1 01111 102 0, -1,+1 11000 220 021 -1,+1, 0 10000 101 0, -1, 0 11001 221

Table 1

In the case four phases PH are possible, the excitation signal can be presented by Table 2 as presented below

PH	EXCITATION SIGNAL
0	T, 0, 0, 0, T, 0, 0, 0, T, 0, 0, 0
1	0, T, 0, 0, 0, T, 0, 0, 0, T, 0, 0
2	0, 0, T, 0, 0, 0, T, 0, 0, 0, T, 0
3	0, 0, 0, T, 0, 0, 0, T, 0, 0, 0, T

Table 2

15

10

In Table 2 the letter T represents a ternary value (-1, 0, +1) according to Table 1. As stated before, the excitation signals are subsequently generated by a ternary generator. If the mean square error for a particular codebook entry generated by the ternary generator is

10

15

20

lower than the mean square error tried before this codebook entry, the ternary count value is temporarily stored in a buffer memory. When all codebook entries have been tried, the buffer memory holds the best ternary count value.

From this count value the codebook inverter 29 derives the binary representation to be used for transmission. It is observed that the most right bit of the binary representation according to Table 1 is the least vulnerable, because an error in it causes the ternary value to change only by +1 or -1 at one position.

The codebook according to Table 1 has the property according to an aspect of the invention that the binary representation of a first codebook entry $G(i_1)$ is equal to a binary sequence of symbols $B(i_2)$ representing a second codebook entry $G(i_2)$, and that the binary representation of said second codebook entry $G(i_2)$ is equal to the binary sequence of symbols $B(i_1)$ associated with the first codebook entry $G(i_1)$. This property can be utilized for enabling the use of the same table (or algorithm) for encoding and decoding the codebook entry.

If e.g. the ternary value $G(i_1) = 122$ in Table 1 is the best codebook entry, the decimal value associated to it is $1 \cdot 3^2 + 2 \cdot 3^1 + 2 \cdot 3^0 = 17$ (decimal). The binary representation of 17 (decimal) is 10001. Using this binary value $B(i_2)$ to address Table 1, a corresponding ternary value $G(i_2)$ of 100 is found. The binary value corresponding to 100 (ternary) is 01001, being equal to the binary value $B(i_1)$ corresponding to the codebook entry with ternary value 122.

The codebook inverter uses the above mentioned property to determine the sequence of symbols to be transmitted. It only needs the function $B(i)\Pi G(i)$, a function which is also needed in the decoder. Consequently this function can be shared between an encoder and a decoder in a full duplex terminal comprising a transmitter and a receiver.

	G(i)	B(i)	G(i)	B(i)	G(i)	B(i)	G(i)
00000000	00000	01000000	02121	10000000	11202	11000000	21210
00000001	00001	01000001	02120	10000001	11212	11000001	21211 .
00000010	00002	01000010	02110	10000010	11211	11000010	21212
00000011	00012	01000011	02111	10000011	11210	11000011	21202
00000100	00011	01000100	02112	10000100	11220	11000100	21201
00000101	00010	01000101	02102	10000101	11221	11000101	21200
00000110	00020	01000110	02101	10000110	11222	11000110	21100
00000111	00021	01000111	02100	10000111	10222	11000111	21101
00001000	00022	01001000	02200	10001000	10221	11001000	21102
00001001	00122	01001001	02201	10001001	10220	11001001	21112
00001010	00121	01001010	02202	10001010	10210	11001010	21111
00001011	00120	01001011	02212	10001011	10211	11001011	21110
00001100	00110	01001100	02211	10001100	10212	11001100	21120
00001101	00111	01001101	02210	10001101	10202	11001101	21121
00001110	00112	01001110	02220	10001110	10201	11001110	21122
00001111	00102	01001111	02221	10001111	10200	11001111	21022
00010000	00101	01010000	02222	10010000	10100	11010000	21021
00010001	00100	01010001	12222	10010001	10101	11010001	21020
00010010	00200	01010010	12221	10010010	10102	11010010	21010
00010011	00201	01010011	12220	10010011	10112	11010011	21011
00010100	00202	01010100	12210	10010100	10111	11010100	21012
00010101	00212	01010101	12211	10010101	10110	11010101	21002
00010110	00211	01010110	12212	10010110	10120	11010110	21001
00010111	00210	01010111	12202	10010111	10121	11010111	21000
00011000	00220	01011000	12201	10011000	10122	11011000	22000
00011001	00221	01011001	12200	10011001	10022	11011001	22001
00011010	00222	01011010	12100	10011010	10021	11011010	22002
00011011	01222	01011011	12101	10011011	10020	11011011	22012
00011100	01221	01011100	12102	10011100	10010	11011100	22011
00011101	01220	01011101	12112	10011101	10011	11011101	22010
00011110 .	01210	01011110	12111	10011110	10012	11011110	22020
00011111	01211	01011111	12110	10011111	10002	11011111	22021
00100000	01212	01100000	12120	10100000	10001	11100000	22022
00100001	01202	01100001	12121	10100001	10000	11100001	22122
00100010	01201	01100010	12122	10100010	20000	11100010	22121
00100011	01200	01100011	12022	10100011	20001	11100011	22120
00100100	01100	01100100	12021	10100100	20002	11100100	22110
00100101	01101	01100101	12020	10100101	20012	11100101	22111
00100110 -	01102	01100110	12010	10100110	20011	11100110	22112
00100111	01112	01100111	12011	10100111	20010	11100111	22102
00101000	01111	01101000	12012	10101000	20020	11101000	22101
00101001	01110	01101001	12002	10101001	20021	11101001	22100
00101010	01120	01101010	12001	10101010	20022	11101010	22200
00101011	01121	01101011	12000	10101011	20122	11101011	22201
00101100	01122	01101100	11000	10101100	20121	11101100	22202

10

15

00101101	01022	01101101	11001	10101101	20120	11101101	22212
00101110	01021	01101110	11002	10101110	20110	11101110	22211
00101111	01020	01101111	11012	10101111	20111	11101111	22210
00110000	01010	01110000	11011	10110000	20112	11110000	22220
00110001	01011	01110001	11010	10110001	20102	11110001	22221
00110010	01012	01110010	11020	10110010	20101	11110010	22222
00110011	01002	01110011	11021	10110011	20100		
00110100	01001	01110100	11022	10110100	20200		
00110101	01000	01110101	11122	10110101	20201		
00110110	02000	01110110	11121	10110110	20202		
00110111	02001	01110111	11120	10110111	20212		
00111000	02002	01111000	11110	10111000	20211		<u> </u>
00111001	02012	01111001	11111	10111001	20210		-
00111010 -	02011	01111010	11112	10111010	20220		
00111011	02010	01111011	11102	10111011	20221		·
00111100	02020	01111100	11101	10111100	20222		
00111101	02021	01111101	11100	10111101	21222		
00111110	02022	01111110	11200	10111110	21221		<u> </u>
00111111	02122	01111111	11201	10111111	21220		

Table 3

Table 3 comprises 243 codebook entries which are addressed by 8 bits indices. It has the same properties with respect to inverse mapping as the codebook according to Table 1.

It is observed that fixed codebook sequences can be obtained by concatenating the sequences according to Table 1 and Table 3 once or more than once. In this way codebook entries having an arbitrary number of samples, except 1, 2, 4 and 7 samples, can be realized. This is in particular advatageous for multirate coders. The representation of these codebook entries is simply formed by the concatenation of the correponding 5 bit and 8 bit indices.

The excitation parameters I_A , G_A , I_F represented by B(i) and G_F are multiplexed by a multiplexer 44. At the output of the multiplexer 44 the multiplexed signal is available for further encoding by the channel encoder 6 is Fig. 1.

In the source decoder 20, according to Fig. 3, the signal received from the channel decoder 18 (Fig. 1) is applied to a demultiplexer 46. The demultiplexer 46 extracts the prediction parameters LPC and the excitation parameters G_A, G_F, I_A and I_F, the latter being represented by the sequence of symbols B(i).

The adaptive codebook index I_A is applied to an input of an adaptive codebook 50. The output of the adaptive codebook 50 is applied to a first input of a multiplier 54. The

10

15

12.05.1998

adaptive codebook gain G_A is applied to a second input of the multiplier 54. The output of the multiplier 54 is connected to a first input of an adder 58.

The fixed codebook index I_F, represented by the sequence of symbols B(i), is applied to an input of a fixed codebook 52 having codebook entries according to the present invention. The output of the codebook 52 is connected to a first input of a multiplier 56. The fixed codebook gain G_A is applied to a second input of the multiplier 56. The output of the multiplier 56 is connected to a second input of the adder 58. At the output of the adder 58 the excitation signal for a synthesis filter 60 is available. The excitation signal is also applied to an input of the adaptive codebook in which the most recent excitation samples are written and from which the least recent excitation samples are removed.

The synthesis filter 60 derives a synthetic speech signal from the excitation signal available at the output of the adder 58. To do so the synthesis filter 60 receives the LPC parameters LPC from the demultiplexer 46.

In the flow graph according to Fig. 4 the numbered instructions have the following meaning:

Nr.	inscription	meaning
62	BEGIN	The program is started.
64	L:=N; MSD:=M ^{N-1} ; K:=I; G:=0	The running variable L is set to the number of
		excitation samples N. The value of the Most
		Significant Digit (MSD) under consideration is set
		to M^{N-1} . The variable K is set to the index I. The
		intermediate result G is set to 0
66	L≠1?	It is checked whether L differs from 1.
68	QUOT := K DIV MSD;	The variables QUOT and REM are calculated
	REM := K MOD MSD;	from K and MSD.
	G := M*G + QUOT	The intermediate result G is recalculated.
70	ODD(QUOT)?	It is checked whether the variable QUOT is odd.
72	K := MSD - 1 - REM	The new value of the variable K is calculated for
	•	K is odd.
74	K := REM	The new value of the variable K is calculated for
		K is even.
76	MSD:=MSD/QUOT	The new values of L, G and MSD are calculated.
	L := L - 1	
78	G_OUT=QUOT*G+K	The final value G_OUT of the codebook entry is
		calculated.
80	END	The program is terminated.

The program according to the flow graph of Fig. 4 is arranged for calculating the plurality of excitation samples for a given value of the index i. It is observed that the binary representation of i is transmitted. The plurality of excitation samples is represented by an M-ary number G(i,N) of which the digits represent the excitation samples. N is the number of samples and consequently the number of digits in the M-ary number.

The calculation of G(i,N) is based on a recursive definition of G(i,N). If each codebook entry comprises N samples, the codebook can be represented as a set of $L=M^N$ vectors sequences of samples $\mathbf{x_0}$, $\mathbf{x_1}$, $\mathbf{x_2}$, \cdots , $\mathbf{x_{L-2}}$, $\mathbf{x_{L-1}}$. The codebook can be extended by one sample value to N+1 samples, by adding digits to the different vectors according to:

10

15

20

25

30

 $0x_0$, $0x_1$, $\cdot\cdot$, $0x_{L-2}$, $0x_{L-1}$, $1x_{L-1}$, $1x_{L-2}$, $\cdot\cdot$, $1x_1$, $1x_0$, $2x_0$, $2x_1$, $\cdot\cdot$, $2x_{L-2}$, $2x_{L-1}$ (in case of a ternary codebook). For N is equal to 1, the function G(i, N) is equal to i. For i larger than N, i is decomposed into the sum of a quotient q of i and the value M^{N-1} of the N^{th} digit of G, and a remainder r. This decomposition is performed for all values of N for which i is smaller or equal to M^{N} -1. From q the value G(i,N) is calculated according to:

$$G(i,N) = \begin{cases} q \cdot M^{N-1} + G(i - q \cdot M^{N-1}, n - 1); & q \text{ is even} \\ q \cdot M^{N-1} + G((q+1) \cdot M^{N-1} - i - 1, n - 1); & q \text{ is odd} \end{cases}$$
(A)

The program according to Fig. 4 determines the value of G(i,N) in a recursive way from i. The program starts at instruction 62. In instruction 64 an variable L is set to N. The value of the most significant digit MSD is made equal to M^{N-1} . The value of variable K is set to the value of the index i of the function G(i,N) to be calculated. The variable G is set to 0.

In instruction 66 it is checked whether L is unequal to 1. If L is unequal to 1 the calculations are continued with instruction 68. In instruction 68 first the quotient QUOT of K and MSD is determined. This corresponds to the determination of the most significant digit of K. Subsequently the remainder REM of the division of K by MSD is determined. This corresponds to the determination of the value represented by the remaining digits of K. Finally an intermediate value of G is determined by multiplying the previous value of G by M and adding the value of QUOT to G.

In instruction 70 it is checked whether the quotient QUOD is even or odd. In the case QUOD is even, the value of K is made equal to the remainder REM. In the case QUOD is odd, the value of K is made equal to MSD-1-REM. This different way K is calculated for even and odd values of QUOD is caused by the ordering of the values of G as function of the index i. From Table 1 it can be seen that the value of the most significant digit of G but one increases as function of i for even values of the most significant digit of G. The value of the most significant digit of G but one decreases as function of i for odd values of the most significant digit of G.

In instruction 76 first the value of MSD is divided by M in order to prepare for the repetition of the previous calculations for the most significant digit of I but one. Subsequently the value of L is decremented and the program is continued at instruction 66. In this way all digits of I are converted to the codebook entry represented by G. If L is equal to 1, the process of converting is finalized, and in instruction 78 the final value of G is calculated by multiplying the value of G found by the previous calculations by M and adding the value of K. In instruction 80 the program is terminated.

Before the codebook entry calculated according to the above program is applied to a synthesis filter it has to be converted into an M-ary representation. As mentioned before, the algorithm according to the program shown in Fig. 4 can also be used to find the index i from a given codebook entry. In order to do so, the program has first to be called with the codebook entry as input. Subsequently the program has to be called again but now with using the result of the first call of the program as input. The index i is now found by converting the result of the second call of the program into a binary number.